Lessons learned from AQR: Essential elements of the model review process

Received (in revised form): 18th November, 2014

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Abstract This paper focuses on the steps performed during an external review of processes and methods related to the valuation of derivatives. The review covers the adequacy of the chosen models, and the models' parameterisation and calibration, as well as implementation features, followed by a discussion of risk-mitigation and risk-monitoring techniques. We illustrate each step with practical examples. The purpose of such a review of derivative pricing models is to identify potential gaps in order to improve the existing modelling framework. The external review is not part of the regular model validation process, but relies largely on the available model validation documentation. Hence, the paper addresses primarily risk managers responsible for the embedding of the above-mentioned steps in the model validation documentation. Nevertheless, auditors and model reviewers might also be interested in the discussion of these review steps. Other organisational aspects (eg the allocation of responsibilities, escalation procedures, etc) are beyond the scope of this paper.

Keywords: *model review, risk mitigation, risk monitoring*

INTRODUCTION

The aftermath of the financial crisis has taught us — among many other lessons — that restructuring and better supervision in the risk management area is deemed necessary because Europe's credibility in the financial services industry is, for society at large, imperative for the good functioning of the economy. With many regulatory changes, such as Basel III, CRD IV,[1] EMIR,[2] Solvency II,[3] etc, the financial sector has shaped itself in a post-financial crisis era into a more risk responsible one. The following questions arise, however: Is the European banking system trustworthy and transparent? Is it able to sustain development in the real economy?

In November 2014, in order to restore confidence in the European banking sector, national governments transferred the role of direct supervision of the significant[4] European banks to the ECB via its newly created institution, the Single Supervisory Mechanism (SSM). In order to line up and prepare for this takeover, the ECB performed a tedious process known as a 'comprehensive assessment'.

The transfer process includes many aspects, one of which is a review of current practice in the impacted banks. As part of the comprehensive assessment, the Asset Quality Review (AQR) addresses the revision of multiple processes within the bank. This article highlights one of these processes, namely the derivative pricing model review, and gives a general overview of the major steps involved therein.

Model review is a core process in any regulatory audit and gains more and more ground from a methodical and procedural point of view. The scope of this article is to discuss various aspects of the model review process and to outline the crucial factors for its success.

Model review is done on the basis of existing documentation. The purposes of the model review are:

- verification of the model adequacy and, in case of known model deficiencies, verification of the appropriateness of the reserves kept against those deficiencies;
- verification of the efficiency of numerical implementation, taking into account the complexity of the payoffs applicable to the products priced with the enquired model;
- verification of the model parameterisation, particularly the existence and consistency check of the liquidity of the market data needed for model calibration;
- investigation into the stability and efficiency of model calibration;
- approval of the existence and sufficiency of the reserves covering the model deficiencies.

Thus the completeness of the existing documentation can significantly facilitate a model review pursued by an independent third party. Most likely, such documentation is written within the scope of the initial model validation. At this point, we would like to point out the fact that model validation is a detailed and extensively quantitative procedure, based on which an efficient and market-conforming evaluation of the enquired products takes place. Within a model validation process, all the aforementioned steps are generally taken into account, estimated and documented. The extent of the detail is in far greater depth than that of a model review process, however.

Besides reviewing the documentation, the scope of the model review can be extended by revaluing some selected test products, using a benchmark such as an external pricing library.

The following paragraph focuses on important topics relating to model review, ie:

(a) model adequacy and implementation;
(b) model parameterisation and calibration;
(c) reserves held against model uncertainties/deficiencies.

Since each of these discussion points is related to some uncertainty, we conclude with some techniques for the proper monitoring and mitigation of the risk factors behind such uncertainties.

MODEL REVIEW

Model adequacy and implementation

In general, an independent expert cannot be assumed to be familiar with the specific scheme and nomenclature used to classify the various derivatives within a particular bank. In order to make a decision as to whether the chosen model is adequate as well as correctly implemented, what are the steps that the expert must go through?

As a first step, the products, as well as the product features valued by the model, need to be analysed. The aim of this step is to determine the minimal model features that are needed for proper product pricing and risk measurement. For this purpose, one needs to identify the driving factors of the products. For each product, the asset component(s) (equity, interest rate, etc) involved within the payoff need to be identified. Following this, the appropriate number of driving risk factors has to be determined for each asset component. The number of appropriate driving factors strongly depends on the product details and features, and consists of the following elements:

- number of driving factors for asset modelling;
- number of driving factors for modelling of asset volatility.

The model should either contain at least one risk factor per involved asset class, or the asset dependency should be neglected and not modelled.

An increase in product complexity tends to increase the number of required driving factors due to rising requirements for the appropriate market dynamic. Interest rate products depend on the evolution of the discount curve (and maybe additional tenor or basis spread curves). Complex products require joint modelling of the curve evolution at different time buckets and may enforce multiple factors. The presence of optionality within the product enforces pricing consistent with observable quotes for related options, which might be used for hedging. Hence, modelling asset volatility has to be considered, especially if matching the volatility smile or the volatility skew observed in the market is a concern. A common way to achieve this implies increasing the number of driving factors, with volatility becoming a stochastic variable, ie a driving factor itself. Another source of complexity is correlation dependency. In general, correlation parameters are not directly observable and, since typical hedging instruments depend only weakly on correlation, calibration to these liquidly traded instruments is rarely possible. Therefore, correlation parameters often have to be fitted to historical data.

The following two examples illustrate the aforementioned stylised facts:

- *Example 1 (exotic equity option)*. What is the appropriate extension of the Black Scholes equation in order to evaluate exotic equity options? Within recent literature, one can identify two major ideas for the pricing of exotic equity options: the local volatility model using the Dupire approach for short maturities and the stochastic volatility extension to provide us with smile dynamics for long maturities. While both models are employed in any options market in which the underlying's volatility is predominantly a function of the level of the underlying, the first approach is more useful when the exotic option has a short maturity; the latter one is predominant in order to provide smile dynamics for longer maturities of the exotic option. Hence, depending on the maturity of the products to be priced, the used model should to some extent include one or both model features at the same time.
- *Example 2 (constant maturity swap [CMS] spread option)*. Depending on two CMS rates, an appropriate model for this product that is consistent with the valuation of hedging instruments requires joint modelling of both rates with a model covering the full interest rate dynamics over a long time horizon. The correlation between the two rates is usually calibrated to CMS spread options, which are available in the market. Since this option depends on the difference of the two CMS rates, accurate volatility modelling is key. If the volatility smile is modelled by one additional stochastic variable for each rate, we end up with two risk factors for each rate, which requires making assumptions about the correlation matrix, since it could not be fully implied by the market. These assumptions would need to be approved by senior management and be well documented.

The construction of a complex model should be reviewed critically, though. It is reasonable to include the material risk factors only. This allows one to increase the performance of the valuation process and helps avoid over-fitting problems by decreasing the model parameters to a reasonable number.

Incorporation of all necessary model features is only one criterion an adequate model needs to fulfil. The second aspect focuses on the model behaviour in extreme situations, such as if the underlying is far in or out of the money. Short and long maturities should also be investigated. Moreover, it has to be noted that implementation failures may cause a more significant mispricing than less important and therefore neglected risk factors would (such as, for example, smile dynamics). It is therefore necessary that the provided documentation describes the investigation pursued within the aforementioned extreme conditions and that possible mispricing is managed from a materiality point of view.

The next step should include the review of the documentation of the initial model validation, which normally takes place in the scope of a new product process (NPP). In particular, comparing the used model with the model requirements which were established during the first aforementioned validation phase should be properly documented. If the used model disregards one of the initially established requirements, this will lead to essential differences in the valuation. The mismatch — depending on its materiality[5] — needs to be mentioned within the model validation documentation, where a comparison of different models should be included. If alternative choices are possible, the practical reasoning for the resulting model choice (after/within the model validation phase) should be stated explicitly within the documentation, also taking into account risk mitigation actions. Furthermore, the frequent model validation cycles should provide further comparison of considered models in order to detect the necessity of model enhancements in time. At this stage of the model review process, the independent third party reviewing the model could use an alternative model implementation, be it a vendor or an own model, to estimate the appropriateness of the model selection from a quantitative point of view.

If the model is adequate to price the enquired products, the correctness and efficiency of the model implementation needs to be challenged. This step is driven by product requirements and takes into account the necessary performance. Therefore, models designed for certain products or (semi-)analytical solutions are often preferable to generic numerical simulations. All assumptions and calculation steps should be carefully reviewed and, if possible, reproduced. This approach could usually only be applied to a limited number of payoffs. For most complex products, numerical methods have to be used. Here, the review should focus on the appropriate discretisation and adequacy of the numerical methods used. For example, the evaluation of a barrier option based on the Monte Carlo (MC) approach requires the use of Brownian Bridges to estimate the probability of touching the barrier between two discretisation points. For any numerical methods used, the essential aspects of the numerical implementation in place should be discussed and documented. The results of this analysis are expected to be made available to support the choice of the numerical method, numerical schemes and their implementation. Besides that, the efficiency of the implementation and convergence of the numerical methods employed should be a part of this analysis. The presence of academic research on this topic and the references provided can help to validate the appropriateness of the numerical implementation.

Having reviewed the model adequacy and implementation, the next stage is to review the parameterisation and calibration of the enquired model.

Model parameterisation and calibration

Most models need to be calibrated. For the calibration of the model parameters an appropriate set of liquid[6] market data is necessary. The following points are important for model parameterisation:

- selection of the market dataset for an appropriate evaluation of the requested products;
- quoting conventions of used market data;
- employment of smoothing and transformation methodologies;
- using proxies: replacement of ideal, but illiquid, benchmark quotes by next best liquidly traded fit;
- estimation of non-observable parameters.

Reviewing the selection of appropriate market data for the model is — independent of the appropriateness of the model — nowadays an essential part of an adequate valuation of derivatives. It should be kept in mind that the same model can be parameterised by different sets of market data. Hence, the choice of market data for model parameterisation and calibration depends on the products that are being valued.

- *Example 3*. Consider the pricing of interest rate products with the Libor market model. For the pricing of autocaps, the parameterisation should take into account prices of caps, whereas the parameterisation for the pricing of callable CMS products should include prices of swaptions and CMS spread options.

Even if all market data are suitable for the model parameterisation and based on liquidly traded instruments, the form in which the data can be used for the model parameterisation should be carefully analysed. For example, conventions for swaption quotes, which are an essential input for interest rate model calibration, changed over time: since the beginning of the financial crisis in 2008, swaptions are quoted using a framework with two distinct curves for discounting and forward rate calculation, but some banks still continue using a single curve dynamic for model calibration. Therefore, an appropriate approach to transforming the volatility surface or cube should be mandatory. Furthermore, after the advent of negative rates in the market, different quotations (eg based on displaced diffusion models and normal models) are available in the market at the same time. Thus, an approval of the arbitrary data sample used by banks should consider a comparison with appropriate datasets from data providers within the scope of the model review.

Often, raw market data are also unsuitable for model parameterisation. For example, equity option quotes have to fulfil additional numerical requirements before the application of the Dupire formula yields valid results. In this context, the application of smoothing algorithms to transform raw market data is a common practice among market participants. This smoothing algorithm can be done in different ways. Within the market, however, common expectations of the quality of smoothed data have been established:

- There are no arbitrage possibilities induced by using smoothed data.
- Smoothed data are located within bid/ask margins.
- Smoothed data prohibit a proper behaviour in limit situations.

Generally, research regarding the smoothing approach should be properly documented to support the bank's choice of a particular smoothing method by a comparison of different methods. Smoothing effects could be quantified by means of some test cases, especially for extreme situations (short maturities, out of the money position, and so on).

Often, the required market data for the model parameterisation are only partially liquid or available. For example, many models use European option prices as benchmarks for the model calibration. For some asset classes, such benchmark instruments are not liquidly traded or their prices are not available at an appropriate frequency, eg daily. In such a case, they might be replaced by alternative instruments. For instance, these new benchmark instruments can have a different exercise type, but will still be treated as European options. Such a set of the instruments is often used for the calibration of equity or commodity models: for some underlyings, American options are available in the market, whereas European options are illiquid. Such treatment of data can only be implemented if the early expiration premium of the American option is negligible. The research on this issue should be properly evaluated. Here, a regular analysis is expected to be pursued, since at some market phases the discrepancies can become significant.

In many cases, some of the components for the model parameterisation are still not available and need to be estimated or determined by experts' guess. A prominent example of this type of data is given by correlations, which are generally not directly observable and are often difficult to imply from liquidly traded instruments. There are two different approaches to obtaining a correlation value. For some asset classes, correlation-dependent products, eg spread options, are traded. The correlation, ie implied correlation, can be inferred from market prices. This method is usually not robust, since correlation is mostly less sensitive to the price changes of used market data. Alternatively, the correlation can be estimated on historical data. Such historical correlation often differs from the implied correlation that is needed as model input. Therefore, the appropriate choice of correlation needs to be discussed and documented.

The calibration procedure is applied to match the (theoretical) model outputs/prices to market quotes (or the used proxies) by varying model parameters. Efficient and stable calibration is a critical aspect of the adequate model selection process. There are a few issues which need to be emphasised:

- For calibration purposes, the theoretical model payoffs as well as benchmarks are frequently represented in an analytical, tractable but only approximately correct form in order to improve the calibration speed. Since the enquired products are usually estimated using numerical methods, it is necessary to re-evaluate the benchmarks using the same numerical method. Benchmark prices are expected to be invariant with respect to the evaluation method used. In order to monitor the adequacy of the calibration, it is useful to take some benchmarks out of the calibration procedure (calibration dataset) and use them as a test portfolio. Doing so, the prices of the benchmark instruments from the test portfolio are expected to be close to the corresponding market quotes. Using this method, one can demonstrate that over-fitting does not occur with the chosen combination of model and calibration method.
- The dependence of benchmark prices on input model parameters can be quite complex in the sense that the calibration problem is not convex. The existing documentation should describe how the initial parameter guess affects the calibrated parameters. If the initial parameter guess strongly influences the resulting calibrated parameters, a variable transformation might be deemed necessary for a substantial improvement of calibration stability.
- Last but not least, an important aspect of the appropriateness of the calibration method is the stability of the calibrated parameters over time. Such analysis is usually performed during the initial validation either on historical data or as daily monitoring of the implemented models.

Together, all of the previously mentioned points — the benchmark evaluations, the initial parameter guess, as well as the calibration stability — are essential criteria to clarify whether the calibration method applied is efficient and thoroughly implemented.

Let us assume that the selected model has undergone all the aforementioned review steps and is found to be adequate according to the documentation. Still, all models have some kind of deficiencies. These model deficiencies might have different origins, but all of them are to be seen as potential risks[7] which need to be mitigated. The following paragraph discusses how to verify the magnitude of the estimated model reserves and how to accumulate them.

Reserves held against model uncertainties/deficiencies

Model deficiencies can have different origins: known potential risks can be reduced via hedging,[8] whereas for other, unsecured risk factors, regulatory requirements demand the building of model reserves. For complex derivatives, there are different sources that one needs to take into account:

- Choosing one model over another always bears the risk of pricing off-market. Further issues are adjustments (eg CMS adjustments), simplifications or assumptions made for (semi-)analytical valuation. Thus, a periodical comparison, eg quarterly, between similar models can provide an estimation of the required reserve amount. For this purpose, a model from a third-party vendor or alternative model could be used. Due to technical and performance considerations, the simplest procedure is preferred.
- The valuation uncertainties due to unobservable parameters, the use of proxies and transformations or smoothing methods are other potential sources of risks. For example, the uncertainty of some correlation — an unobservable parameter — estimate can be evaluated by calculating the maximum historical variance of the correlation over different time windows. The reserve amount for this particular parameter can then be estimated as the product of the sensitivity of the enquired product portfolio against correlation and the correlation uncertainty. The usage of special proxies, especially those that are not widely used by market participants, should be extensively compared with alternative methods. The same holds for both transformation and smoothing methods.
- Numerical errors (MC error, discretisation uncertainty, etc), extrapolation, and interpolation

uncertainties need to be mitigated by model reserves, too.

After estimating all the single reserves for different model deficiencies, these single reserves need to be aggregated. An appropriate aggregation approach is to assume that all risk factors have a similar probability distribution around the mean value. Then, assuming no correlations between the risk factors, the total uncertainty of the joint distribution can be calculated. This approach yields a total reserve that is equal to the square root of the sum of the squared single uncertainties.

CONCLUSION

The steps illustrated are characteristic of any review process for derivative pricing models. The experience gained from AQR shows that the anchorage of the aforementioned analyses and observations into the already existing model documentation facilitates the review process for both auditors and risk managers. The embedding of the previously mentioned analyses should ideally occur at the earliest possible time within the model lifecycle. One possible approach is the enforcement of such considerations within the model validation policy. This approach automatically improves the existing modelling framework and highlights the deep understanding of a bank for derivative modelling and the related model risk.

References and notes

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2 EU Banking and Finance, 'Derivatives/EMIR', available at: http://ec.europa.eu/internal_market/financial-markets/derivatives/index_en.htm (accessed 18th February, 2015).

3 European Insurance and Occupational Pensions Authority (EIOPA), 'Solvency II', available at: https://eiopa.europa.eu/en/activities/insurance/solvency-ii/index.html (accessed 18th February, 2015).

4 European Central Bank, 'Who supervises my bank?', available at: https://www.bankingsupervision.europa.eu/banking/list/who/html/index.en.html (accessed 18th February, 2015).

5 The model should be rejected if the mismatch is classified as material.

6 The liquidity of the market data can be also part of the model review. We consider this to be a separate topic that lies beyond the scope of this paper.

7 Accounting reserve requirements, such as bid/ask adjustments or close-out reserves, are not considered here.

8 We remark that hedging methodology is not necessarily a risk mitigation technique in the intended sense.